Exercise 2.6: invariant density

Here we are presenting an exercise for the section 2.8 in the book. We study the invariant density by changing the initial distribution. Exercise: Change the initial distribution in Code 2.8 (Logistic map: density, M trajectories, initial distribution: uniform in (0,1)).

In the code below, the initial points are selected uniformly in the interval [0.5,0.6]. Notice that the code is the same as Code 2.8 in which the initial distribution is uniform in [0,1]. Now the line for the initial points is xinit0[l]<- runif(1,0.5,0.6).

```r
# Logistic map: density, M trajectories, initial distribution: uniform in (0.5,0.6)
f.x<- function(x,r){
r*x*(1-x)
}
r<- 4
M<- 1000 # number of trajectories
nstep<- 25 # number of iterations
xt<- numeric()
xinit0<- numeric()
xens<-matrix(,M,nstep) # to memorize the single trajectory
set.seed(1) # seed of the sequence of (pseudo) random numbers
for(l in 1:M){ # starting loop on the trajectories
  # it is possible to change the initial distribution
  xinit0[l]<- runif(1,0.5,0.6) # uniform distribution in [0.5,0.6]
  x<- xinit0[l]
  x<- runif(1,0.5,0.6)
  xt[l]<- x
  for(i in 1:nstep){ # starting loop on the iterations
    y<- f.x(x,r)
    x<- y
    xt[i]<- x
    xens[l,i]<- xt[i]
  } # ending loop on the iterations
  } # ending loop on the trajectories
mstep1<-1
mstep2<-2
mstep3<-3
mstep4<-5
mstep5<-nstep
lbin<- 0.02
windows()
par(mfrow=c(3,3),cex.main=0.8)
hist(xinit0,probability=T,xlab="x(t)",ylab="Density",main="Initial distr.",
```
The result is reported in Fig. 1. Notice the different scales in the histograms with $t = 1$ and $t = 2$. In the present case, we see that the convergence to the Beta$(0.5, 0.5)$ is slower with respect to the initial density uniformly distributed in $[0, 1]$ (Fig. 2).

We can further chance the initial condition with the instruction:

```r
xinit0[1] <- rnorm(1, 0.5, 0.05)  # normal distribution
```

The initial points are sampled from a normal distribution with mean = 0.5 and standard deviation = 0.05. The result is reported in Fig. 2. Notice the different scales in the histograms with $t = 1$ and $t = 2$. Also in this case, the
Figura 2 Evolution of density of an ensemble of 1000 trajectories of the logistic map with $r = 4$. The initial density is a normal with mean = 0.5 and standard deviation = 0.05.

convergence to the $Beta(0.5, 0.5)$ is slower with respect to the initial density uniformly distributed in $[0, 1]$ (Fig. 2.30). The reader can vary the initial condition and also the value of $r$ in the logistic map.